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# Transient conjugated heat transfer in pipes involving two-dimensional wall and axial fluid conduction

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**Abstract**

This paper presents an analysis for an unsteady conjugated heat transfer problem in thermally developing laminar pipe flow, involving two-dimensional wall and fluid axial conduction. The problem is solved numerically by a finite-difference method for a thick-walled, infinitely long, two-regional pipe which is initially isothermal with a step change in the constant outside temperature of the heated downstream section. A parametric study is done to analyze the effects of four defining parameters, namely the Peclet number, wall-to-fluid thermal conductivity ratio, wall-to-fluid thermal diffusivity ratio and wall thickness to inner radius ratio. The predicted results indicate that, although the parameters affect the heat transfer characteristics at the early and intermediate periods, the time to reach the steady state does not change considerably. With the boundary conditions of the present problem, the thermal inertia of the system is mainly dependent on the flow conditions rather than on the wall characteristics. 

## 1. Introduction

Analysis of unsteady conjugated heat transfer is important for control of heat exchangers during startup, shutdown or any change in the operating conditions. The transient behavior of heat transfer in laminar internal flow has been investigated by many authors under step or periodic change of either boundary or fluid inlet conditions. Early investigators have considered extremely thin walls where wall conduction effects are neglected and conditions at the outer surface of the pipe or duct prevail along the inner surface. In problems that are referred to as conjugated, the thermal boundary conditions along the solid–fluid interface are not known a priori, and the energy equations should be solved under the conditions of continuity in the temperature and/or heat flux.

Conjugated heat transfer problems in laminar duct flow under steady-state conditions have been studied for several geometries and for different boundary conditions since the 1980s and a summary of the literature survey for the related subject may be found in [1].

The first investigator on the unsteady conjugated problem was Succec [2], who studied heat transfer to slug flow between parallel plates with time-varying inlet fluid or duct wall temperatures. Krishan [3] worked on fully developed pipe flow with a step change in heat flux or outer surface temperature. Succec and Sawant [4] suggested an improved analytical method for the conjugated transient heat transfer problem of a parallel plate duct with periodically varying inlet fluid temperature. Succec [5,6] extended the same problem to duct walls exposed to suddenly changing ambient fluid temperature. Cotta et al. [7] studied slug flow inside parallel plate channels and in circular ducts and solved the problem analytically for periodic variation of inlet temperature. Lin and Kuo [8] considered a step change in uniform wall heat flux over a finite length of a long circular duct and solved the problem numerically for the thermal entrance region. The same problem with constant outside surface temperature was studied by Yan et al. [9]. Analytical methods were used by Travelho and Santos [10] for a parallel plate duct with slug flow and varying inlet temperature and by Olec et al. [11] in fully developed pipe flow. Recently, numerical methods were used for thick-walled pipes, considering two-dimensional wall conduction, by Schutte et al. [12], with simultaneously or thermally developing laminar flow under constant wall heat flux; by Lee and Yan [13], with fully developed flow under constant outside surface

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**Nomenclature**

| | |
|---|---|
| $a$ | constant of discretization equation (Eqs. (6a)–(6g)) |
| $c_p$ | specific heat at constant pressure |
| $d$ | wall thickness |
| $k$ | thermal conductivity |
| $Nu$ | Nusselt number |
| $Pe$ | Peclet number |
| $q$ | heat flux |
| $r$ | radial coordinate |
| $t$ | time |
| $T$ | temperature |
| $T_0$ | initial temperature of the system |
| $T_1$ | outer surface temperature of the downstream section for $t > 0$ |
| $u$ | velocity |
| $x$ | axial coordinate |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | thermal diffusivity |
| $\delta r$ | radial position difference |
| $\delta x$ | axial position difference |
| $\Delta r$ | radial step size |
| $\Delta t$ | time step increment |
| $\Delta x$ | axial step size |
| $\rho$ | density |

*Subscripts*

| | |
|---|---|
| b | bulk |
| f | fluid |
| $i, j$ | at nodal point $i, j$ |
| m | mean |
| r | radial |
| s | solid |
| sf | solid to fluid ratio |
| w | at wall–fluid interface |
| $x$ | axial |

*Superscripts*

| | |
|---|---|
| $'$ | dimensionless quantity |
| o | at previous time step |

temperature, and by Yan [14], for thermally developing channel flow with convection from the ambient. In all these final investigations the related boundary conditions were applied to a finite length of the pipe or channel.

## 2. Analysis

In the present problem consideration is given to laminar pipe flow in an infinitely long pipe ($-\infty < x < +\infty$) which has a finite wall thickness, $d$. A schematic diagram of the problem is shown in Fig. 1. At the far upstream ($x = -\infty$) the fluid entering the pipe has a uniform temperature, $T_0$, which is equal to the initial temperature of the system. The upstream side of the pipe ($x < 0$) is sufficiently long so that the flow is hydrodynamically developed at the beginning of the heating section. At time $t = 0$, the temperature of the outer surface of the heated section ($x > 0$) of the wall is suddenly raised to a new value $T_1$ and remains constant thereafter until the system reaches the steady-state conditions. All physical properties of the fluid and wall are constant and viscous dissipation in the flow is neglected.

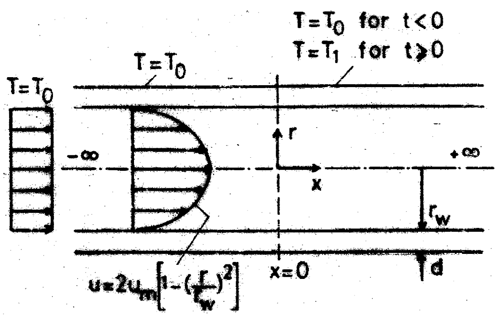


Fig. 1. Schematic diagram of the problem.

The above-described problem may be formulated in non-dimensional form as follows.

In the wall region, the differential equation is

$$\frac{1}{\alpha_{\rm sf}} \frac{\partial T'_{\rm s}}{\partial t'} = \frac{1}{r'} \frac{\partial}{\partial r'} \left( r' \frac{\partial T'_{\rm s}}{\partial r'} \right) + \frac{1}{Pe^2} \frac{\partial^2 T'_{\rm s}}{\partial x'^2}. \tag{1a}$$

The initial and the boundary conditions are:

$$\text{at } t' = 0, \quad T'_{\rm s} = 0; \tag{1b}$$

$$\text{at } x' = -\infty, \quad T'_{\rm s} = 0; \tag{1c}$$

$$\text{at } x' = +\infty, \quad \frac{\partial T'_{\rm s}}{\partial x'} = 0 \quad (T'_{\rm s} = 1 \text{ at steady state}); \tag{1d}$$

$$\text{at } r' = 1 + d' \quad \text{for } x' < 0, \qquad T'_{\rm s} = 0; \tag{1e}$$

$$\text{at } r' = 1 + d' \quad \text{for } x' \geqslant 0, \qquad T'_{\rm s} = 1; \tag{1f}$$

$$\text{at } r' = 1, \quad T'_{\rm s} = T'_{\rm f}; \tag{1g}$$

$$\text{at } r' = 1, \quad \frac{\partial T'_{\rm s}}{\partial r'} = \frac{1}{k_{\rm sf}} \frac{\partial T'_{\rm f}}{\partial r'}. \tag{1h}$$

In the fluid region, the differential equation is

$$\frac{\partial T'_{\rm f}}{\partial t'} + \left(1 - r'^2\right) \frac{\partial T'_{\rm f}}{\partial x'} = \frac{1}{r'} \frac{\partial}{\partial r'} \left( r' \frac{\partial T'_{\rm f}}{\partial r'} \right) + \frac{1}{Pe^2} \frac{\partial^2 T'_{\rm f}}{\partial x'^2}. \tag{2a}$$

The initial and the boundary conditions are:

$$\text{at } t' = 0, \quad T'_\mathrm{f} = 0; \tag{2b}$$

$$\text{at } x' = -\infty, \quad T'_\mathrm{f} = 0; \tag{2c}$$

$$\text{at } x' = +\infty, \quad \frac{\partial T'_\mathrm{f}}{\partial x'} = 0 \quad (T'_\mathrm{f} = 1 \text{ at steady state}); \tag{2d}$$

$$\text{at } r' = 0, \quad \frac{\partial T'_\mathrm{f}}{\partial r'} = 0; \tag{2e}$$

$$\text{at } r' = 1, \quad T'_\mathrm{f} = T'_\mathrm{s}; \tag{2f}$$

$$\text{at } r' = 1, \quad \frac{\partial T'_\mathrm{f}}{\partial r'} = k_\mathrm{sf}\frac{\partial T'_\mathrm{s}}{\partial r'}. \tag{2g}$$

Non-dimensional parameters of the problem are defined as

$$T' = \frac{T - T_0}{T_1 - T_0}, \quad x' = \frac{x}{r_\mathrm{w} Pe}, \quad r' = \frac{r}{r_\mathrm{w}}, \quad d' = \frac{d}{r_\mathrm{w}},$$
$$t' = \frac{t\alpha_\mathrm{f}}{r_\mathrm{w}^2}, \quad k_\mathrm{sf} = \frac{k_\mathrm{s}}{k_\mathrm{f}}, \quad \alpha_\mathrm{sf} = \frac{\alpha_\mathrm{s}}{\alpha_\mathrm{f}} \quad \text{and} \quad Pe = \frac{2u_\mathrm{m} r_\mathrm{w} \rho_\mathrm{f} c_{p_\mathrm{f}}}{k_\mathrm{f}}.$$

Fluid bulk temperatures, $T'_\mathrm{b}$, interfacial wall heat flux, $q_\mathrm{w}$, and local Nusselt numbers, $Nu$, are variables of engineering interest and may be computed as follows:

$$T'_\mathrm{b} = 4\int_0^1 r'(1 - r'^2)T'\,\mathrm{d}r', \tag{3}$$

$$q_\mathrm{w} = -\left.\frac{\partial T'_\mathrm{f}}{\partial r'}\right|_{r'=1}, \tag{4}$$

$$Nu = \frac{2q_\mathrm{w}}{T'_\mathrm{w} - T'_\mathrm{b}}. \tag{5}$$

## 3. Solution methodology

The systems of equations (1a)–(1h) and (2a)–(2g) are solved simultaneously by a numerical finite-difference approach. Eq. (1a) and conductive terms in Eq. (2a) are discretized by central-difference schemes and convective terms in Eq. (2a) are discretized by an exact method defined in [15]. For the transient terms in the equations, a fully implicit formulation in time is applied to assure stability in solutions. The following discretization equation is obtained for a nodal point $(i, j)$ in the fluid region:

$$a_{i,j}T'_{i,j} = a_{i+1,j}T'_{i+1,j} + a_{i-1,j}T'_{i-1,j} + a_{i,j+1}T'_{i,j+1} + a_{i,j-1}T'_{i,j-1} + a^\mathrm{o}_{i,j}T'^\mathrm{o}_{i,j}, \tag{6a}$$

where

$$a_{i+1,j} = \frac{(r'_j - r'^3_j)(\Delta r')_j}{\exp\left[Pe^2(1 - r'^2_j)(\delta x')_{i+1}\right] - 1}, \tag{6b}$$

$$a_{i-1,j} = \frac{(r'_j - r'^3_j)\exp\left[Pe^2(1 - r'^2_j)(\delta x')_{i-1}\right](\Delta r')_j}{\exp\left[Pe^2(1 - r'^2_j)(\delta x')_{i-1}\right] - 1}, \tag{6c}$$

$$a_{i,j+1} = \left[\frac{r'_j}{(\delta r')_{j+1}} + 0.5\right](\Delta x')_i, \tag{6d}$$

$$a_{i,j-1} = \left[\frac{r'_j}{(\delta r')_{j-1}} - 0.5\right](\Delta x')_i, \tag{6e}$$

$$a^\mathrm{o}_{i,j} = \frac{r'_j(\Delta x')_i(\Delta r')_j}{\Delta t'}, \tag{6f}$$

$$a_{i,j} = a_{i+1,j} + a_{i-1,j} + a_{i,j+1} + a_{i,j-1} + a^\mathrm{o}_{i,j}. \tag{6g}$$

Temperature distributions at any time step were found by the line-by-line method [16] visiting the points both in the solid and in the fluid region from outer surface to axis of the pipe and from upstream to downstream. A consecutive procedure is applied in the solutions by transferring information between the solid and the fluid side using the matching conditions of the problem. For a typical run, Eq. (2f) and therefore the previously calculated temperature distribution at the wall–fluid interface is used as a boundary condition for the fluid region. Iteration is then continued in the solid side by condition (1h) and so the interfacial heat flux values are used to transfer information from the fluid to the solid side.

The grids are laid both in the solid and in the fluid side and contracted in radial direction near the interface in both regions. Axial grids are also contracted in the vicinity of the beginning of the heating (or cooling) zone where the temperature step occurs and linearly stretched by taking the axial step size of a grid as 1.5 times the previous grid, increasing both in downstream and upstream directions. The minimum step size used in axial direction is 0.0001 while in radial direction it is $d'/8$. The axial distances both for the downstream and the upstream sides, where conditions (1c), (1d), (2c) and (2d) apply, are estimated by a previous steady-state solution of the same problem [1] and checked by trial runs with coarse grid systems. In the solutions satisfactory results were obtained by using 12 grid spacings in radial direction (four in the solid and eight in the fluid side) and from 35 to 45 grid spacings in axial direction. The number of axial stations and therefore the axial length of the computational region depends on the parameters of the problem and mainly on the Peclet number.

A non-uniform time step is also used to speed up the solutions and to ensure accuracy. The first time step is taken to be 0.0001 and increased by 10% in the subsequent steps.

Accuracy tests were done by increasing the number of grid points, by decreasing the time steps, by changing the locations of the grid points and by reversing the direction of the traversing and sweeping of the points during iterations. The differences between the computed values for any cases were not considerable.

Another indication of the accuracy of the method is the asymptotic time-independent results of the runs. These were compared with the results of [1] and fairly good agreement was seen.

## 4. Results and discussions

Inspection of the analysis shows that the results of the present transient conjugated problem depend on four non-dimensional parameters, namely the Peclet number, $Pe$, wall thickness ratio, $d'$, wall-to-fluid conductivity ratio, $k_{sf}$ and thermal diffusivity ratio, $\alpha_{sf}$. Computations were made for several combinations of these parameters: $Pe = 1$, 5 and 20; $d' = 0.02$, 0.1 and 0.3; $k_{sf} = 1$, 10 and 100 and $\alpha_{sf} = 0.1$, 1 and 10. These values were selected as appropriate for problems of engineering interest and from the range that all the presumed effects of the defined problem, i.e. two-dimensional wall and fluid axial conduction, are in a significant level.

Local Nusselt number as traditionally considered in the presentation of the convection heat transfer results is not a convenient tool for the conjugate problems [17], since it contains three unknowns in its definition. However, local interfacial heat flux gives more useful information. The results, therefore, are presented by local interfacial heat flux values but some results were also given for fluid bulk and inner wall temperature distributions in order to better understand some of the transient behavior of the conjugated problem.

Fig. 2 shows the local interfacial heat flux values for a typical run with average parameter values, for $Pe = 5$, $d' = 0.1$, $k_{sf} = 10$ and $\alpha_{sf} = 1$ at several time steps. Almost the same behavior is observed for the other runs with different combinations of the parameter values.

As can be seen from the figure, there is a substantial amount of heat flux occurring in the upstream region due to the penetration of heat opposite to the direction of flow, resulting from axial wall and fluid conduction. At early transient period heat flux is from wall to fluid, but after a certain time the situation is reversed. At initial times the wall axial conduction is more rapid and inner wall temperatures are higher than the adjacent fluid temperatures. When time increases, the fluid axial conduction increases and the fluid temperatures in the vicinity of the wall, where convection vanishes, are higher than the inner wall temperatures, and therefore negative interfacial heat flux values are obtained. Since the outer temperature of the wall in the unheated upstream side of the pipe is still kept constant for $t > 0$ with its initial value of $T_0$, the pipe wall loses heat outside, resulting in shorter penetration lengths in the wall than in the fluid region, leading to reverse heat flux from the fluid to the wall. The amount of heat flux in the upstream section decreases with increasing upstream distance and increases with time.

In the downstream side of the pipe the curves rise to a maximum value and then decrease, and at early times attain a constant value. At the beginning of the transient period the radial wall conduction is dominant, which results in higher rates of increase in the inner wall temperatures than in the fluid temperatures. As time goes on, heat transfer by convection is more influenced; therefore the value of the peak decreases and the downstream uniformity in heat flux values is disturbed. This trend continues until the system reaches its final steady state.

Figs. 3 and 4 show the axial distribution of the fluid bulk and inner wall temperatures, respectively, for the same run. During the early transient period the down-

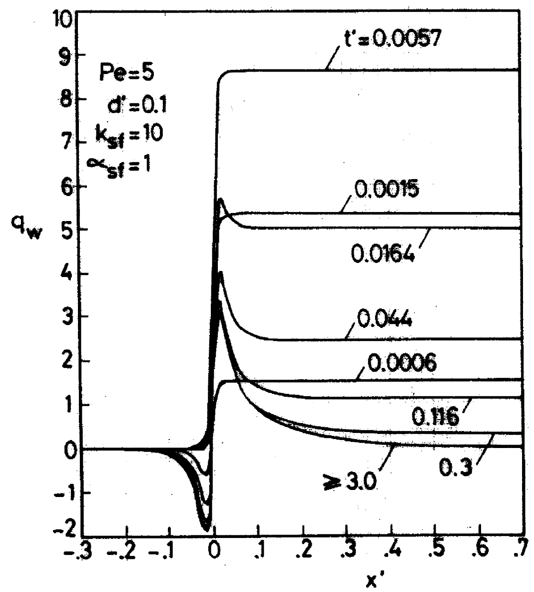


Fig. 2. Transient axial distributions of interfacial heat flux.

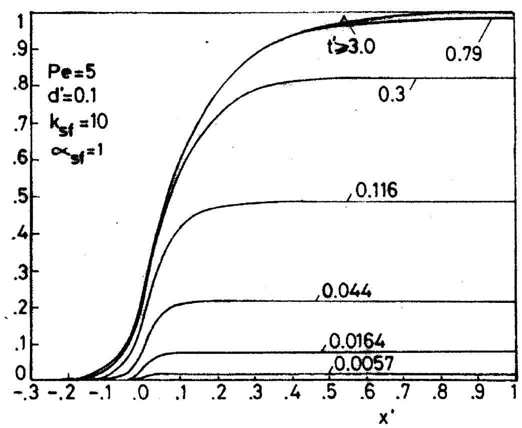


Fig. 3. Transient axial distributions of fluid bulk temperatures.

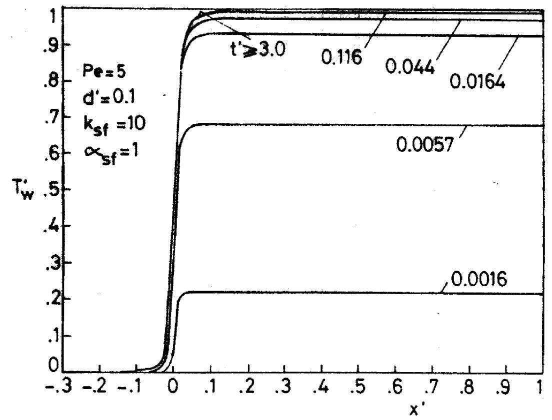


Fig. 4. Transient axial distributions of inner wall temperatures.

stream part of the curves for both $T'_b$ and $T'_w$ are mainly flat as in $q_w$ curves, and the values are rather smaller in the upstream portion. As time increases, the increase in both fluid and wall axial conduction causes an increase in both fluid bulk and interfacial temperatures.

As can be seen from the figures, at early times inner wall temperatures are higher than fluid bulk temperatures and heat flux is from wall to fluid in the upstream portion of the pipe. As time elapses, the penetration of heat backward through the upstream side in the fluid region is more influenced than in the solid side, resulting in higher fluid bulk temperatures compared to the inner wall temperatures. This explains why negative heat flux values are obtained in the upstream section of the pipe. At steady state, both fluid bulk and inner wall temperatures reach their asymptotic value of 1 in the fully developed region.

In order to investigate the effect of pipe wall thickness Fig. 5 is drawn for axial distributions of unsteady interfacial heat flux for different $d'$ values at three different time steps. The thermal resistance and heat capacity of the system are smaller for thinner walls and heat supplied from the outer surface is easily transferred to the fluid. By this reason, for the early transient period, the values of interfacial heat flux are higher for smaller wall thickness. After this early period, when convective heat transfer is becoming more influenced than conduction in the wall, with an increasing effect for thin walled pipes, $q_w$ values are lower for smaller $d'$ values and the curves cross at some distance downstream of the heated section. For thin walls, lower heat flux downstream is due to the increase of $T'_b$ due to early high heat flux and so to small temperature difference $T'_w - T'_b$. Since thick walls exhibit more axial conduction, the extent and the magnitude of reverse interfacial heat flux is smaller and begins later in the upstream section. The time required to reach the steady state is not much affected by the wall thickness and slightly increased for $d' = 0.3$.

Fig. 6 shows the effect of wall-to-fluid thermal conductivity ratio on axial distributions of unsteady inter-

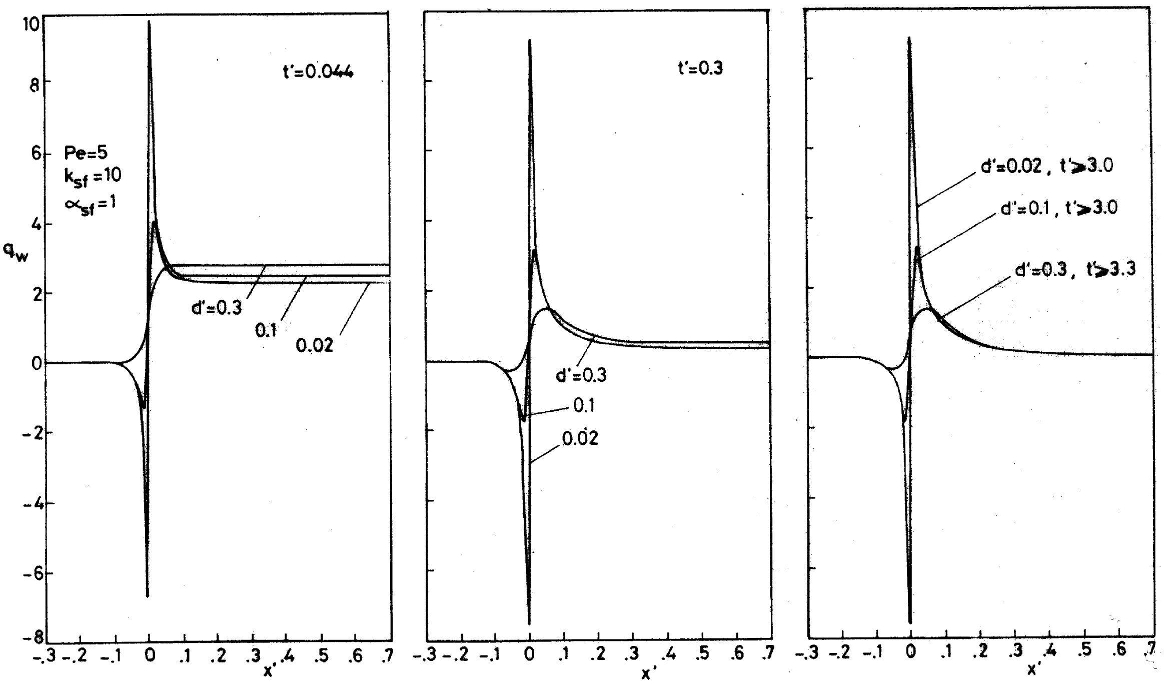


Fig. 5. Effect of thickness ratio on interfacial heat flux.

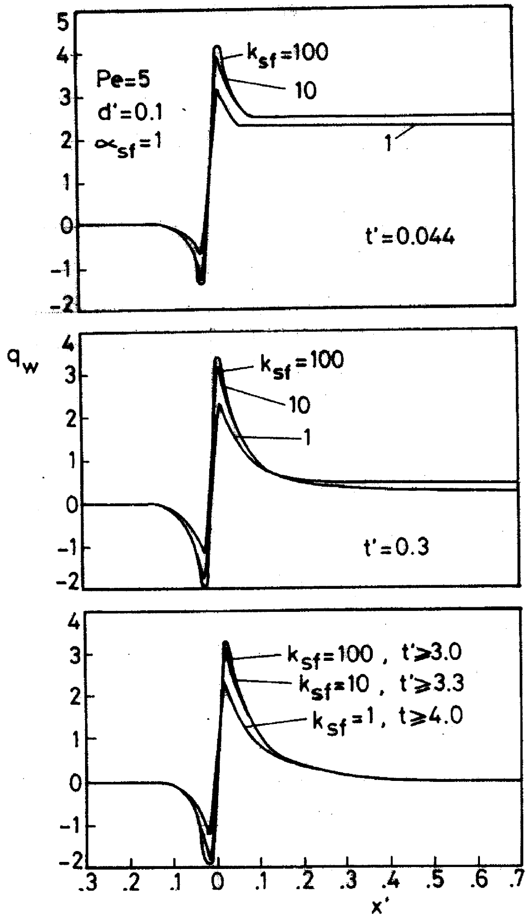


Fig. 6. Effect of conductivity ratio on interfacial heat flux.

facial heat flux. Inspection of this figure reveals that the length of penetration and the amount of reverse heat flux in the upstream section are greater with large $k_{sf}$. Since large $k_{sf}$ means small thermal resistance in the pipe wall, interfacial temperatures and therefore heat flux values are larger in the heated section. The time required to reach the steady state is longer for small $k_{sf}$ values, since small $k_{sf}$ corresponds to small $\alpha_{sf}$, which means large thermal capacity of the pipe wall.

In Fig. 7 comparative results for interfacial heat flux at some instants of time for different values of Peclet number are given. In the upstream section of the pipe both the extent and the magnitude of the reverse heat flux increased with decreasing Peclet number. This is because of greater amounts of axial fluid conduction and penetration of heat backward through the upstream side in the fluid region for small Peclet numbers. The extent of postheating in the downstream section is also increased with decreasing Peclet number, since convective effect is low and therefore the development length is increasing. The degree of peak is smaller and the drop-

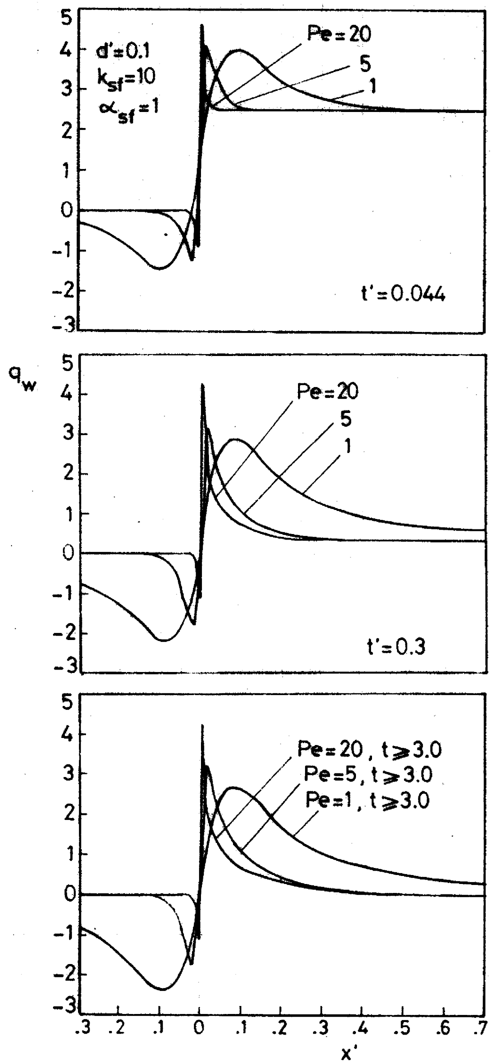


Fig. 7. Effect of Peclet number on interfacial heat flux.

off is more gradual in heat flux values for decreasing Peclet numbers.

Finally the effect of diffusivity ratio, $\alpha_{sf}$, on interfacial heat flux is shown in Fig. 8. At early time periods the values of $q_w$ are higher for large $\alpha_{sf}$. This is because large $\alpha_{sf}$ means small thermal capacity of the pipe wall and the response of $q_w$ is faster. When time elapses, convective heat transfer is dominant over radial pipe conduction with an increasing effect of decreasing $\alpha_{sf}$. This is due to the fact that a lower value of $\alpha_{sf}$ means a higher value of $k_f$ in a relative sense. Therefore after a certain time period $q_w$ values are greater for small values of $\alpha_{sf}$. The development of the curves is faster at initial transient period for the cases of large $\alpha_{sf}$, but the time to reach the steady state is the same for all cases. The final shape of $q_w$ curves is independent of diffusivity ratio as expected.

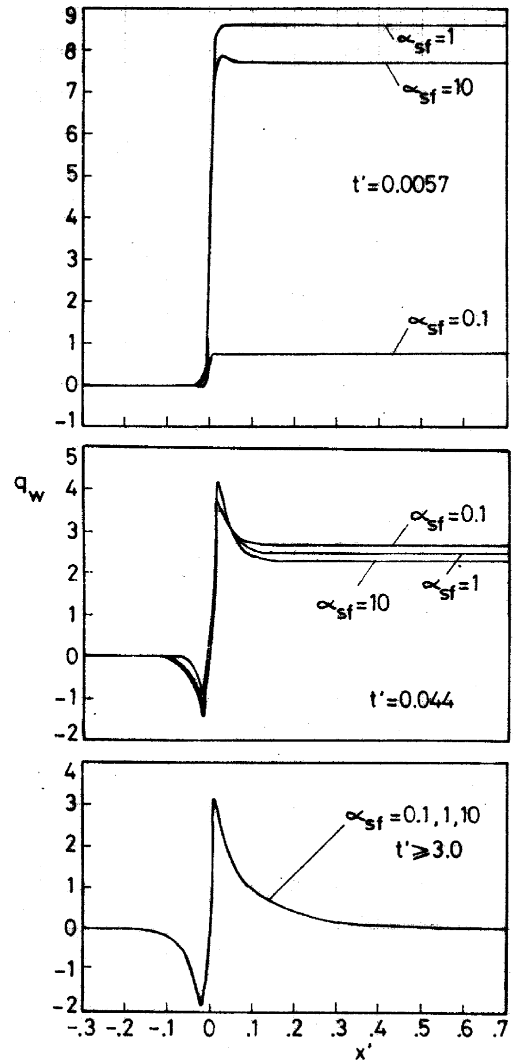


Fig. 8. Effect of diffusivity ratio on interfacial heat flux.

One of the most interesting and surprising outcomes deduced from the present investigation is that, although the unsteady behavior of conjugated heat transfer is considerably affected by the parameters, especially for early and intermediate periods, the time for the system to reach the steady state is not much affected by a single parameter. The cumulative effect of all four parameters, however, is higher, but not as high as expected. Two extreme cases may be considered for comparison. The final time to reach the steady state for the run with $Pe = 1$, $d' = 0.3$, $k_{sf} = 1$ and $\alpha_{sf} = 0.1$ is 8.55 while it is 2.72 for the run with $Pe = 20$, $d' = 0.02$, $k_{sf} = 100$ and $\alpha_{sf} = 10$. This fact is probably due to the boundary conditions of the present problem which are different than those in [12–14], in which the heated portion of the tube is bounded in a finite length and the outer wall is insulated outside of this region. This result indicates that the thermal inertia of the system is mainly dependent on the flow conditions rather than on the wall characteristics.

## 5. Conclusions

In the present paper, a transient conjugated heat transfer problem for thermally developing laminar pipe flow is considered, which takes into account radial and axial conduction both in the wall and in the fluid region. The problem is solved numerically by a finite-difference method and a parametric study is done to investigate the effects of four defining parameters, $Pe$, $d'$, $k_{sf}$ and $\alpha_{sf}$. The results may be summarized as follows.

1. Considerable amount of heat is conducted through the upstream side both in the wall and in the fluid region, and this results in preheating of the fluid before entering the heated section. Due to the outer boundary temperature of the unheated part of the pipe, the wall loses energy to the outside, and the length of backward penetration of heat by axial conduction may be greater in the fluid region. This results in negative interfacial heat flux, i.e. from fluid to the wall in the upstream section of the pipe.

2. In the downstream side of the pipe, interfacial heat flux values rise to a maximum and then decrease. At early transient period they attain a constant value, and as time goes on, heat flux values decrease monotonically until the system reaches the steady state.

3. The effect of wall conduction on heat transfer is more pronounced for high values of $d'$, $k_{sf}$ and $\alpha_{sf}$, and for small values of $Pe$. The effect of the parameters is higher in the early and intermediate transient periods and the time to reach the steady state does not change much with the parameters.

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